XVIII.—*On the Physical Explanation of the Inequality of the two Semi-diurnal Oscillations of Barometric Pressure.—By* Henry F. Blanford, *Meteorologist to the Government of India.*

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There are, perhaps, few phenomena in the domain of terrestrial physics which have received more attention than the diurnal variation of barometric pressure; and on the causes and explanation of which, nevertheless, there is more diversity of opinion even at the present day. Dove, Sabine, Herschell, Espy, Lamont, Kreil, Broun, and many others have in turn engaged in the discussion of this vexed problem; and, at the present time, Mr. Alexander Buchan is publishing an elaborate and most valuable *resumé* of the existing data in the Transactions of the Royal Society of Edinburgh, as a preliminary to a renewed investigation.

The general features of the diurnal variation of pressure are familiar enough to every one who has ever observed the rise and fall of the barometer for a few days in India, and most other tropical countries. From about 3 or 4 in the morning the pressure increases gradually towards sunrise, then more rapidly,—and culminates generally between 9 and 10 A. M. A fall then sets in, which becomes rapid during the hottest hours of the day, and the pressure reaches its minimum generally between 4 and 5 P. M. The pressure then increases till about 10 P. M.; but in general does not attain the same height as at the corresponding morning hour. Lastly, a second fall brings it to a second minimum between 3 and 4 A. M., which, except on mountain peaks and at such stations as Simla and Darjiling, is never quite so low as the afternoon minimum.*

Thus, then, the pressure rises and falls twice in the 24 hours, attaining, in general, its absolute maximum about 9 or 9. 30 A. M., and its absolute minimum between 4 and 5 P. M.

This may be taken as a general description of the phenomenon as exhibited in the tropics; but it presents many striking variations at different

* I must correct this statement. I find, on examining the ship's observations for the month of January recorded in the Bay of Bengal, between N. Latitude 20° and the E. C. Light Ship, *i. e.*, between 60 and 100 miles from the coast, that the form of the diurnal barometric curve afforded by them, in this respect, resembles that of hill stations; the early morning minimum being considerably lower than the afternoon. The relation of this peculiarity to the phenomenon of the diurnal sea-breeze, and the confirmation it affords of the transfer of air from the land to the sea during the daytime, in the strata above that in which the sea-breeze prevails, which is the main topic of this paper, are obvious. I have not as yet obtained the data for other months.—*Note added January 20th,* 1877.

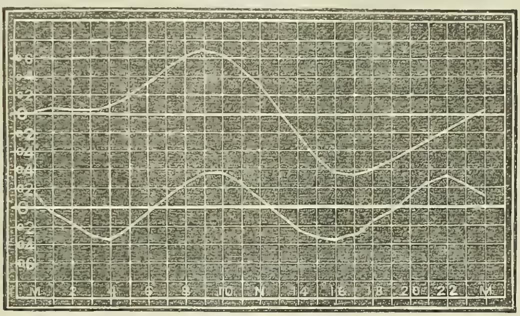
places, and at one and the same place at different times of the year. These variations affect the hour at which the pressure attains its maximum and minimum values, the absolute amplitude of the oscillations, and lastly, their relative amplitude. It is this phenomenon—the variation in the relative amplitude of the day and night oscillations—the probable physical explanation of which I have now to bring to notice.

It was observed by Arago, apparently some years prior to 1841, that in Europe "the proximity of the sea has the effect of diminishing the amplitude of the interval during which the diurnal fall lasts, viz., that which occurs between 9 A. M. and 3 P. M.;" and considering the whole phenomenon as made up of a single and double oscillation, it may easily be shewn that this interval is determined mainly by the relative amplitude of these two elements. The latest notice on the subject is given in the following extract from Mr. Buchan's Memoir, a copy of the first part of which, (for which I am indebted to the author,) has reached me only within the last week.* In summing up the characteristics of the midday fall of pressure, he says:—"Whatever be the cause or causes on which the diurnal oscillations of the barometer depend, the influence of the relative distribution of land and water in determining the absolute amount of the oscillation in particular localities as well as over extended regions, is very great. From the facts detailed, (in Mr. Buchan's paper), it will be seen that this influence gives a strong local colouring to the results, particularly along the coasts; and that the same influence is extensively felt over the Channel, the Mediterranean, the Atlantic, and other sheets of water on the one hand; and on the other over the inland portions of Great Britain, Europe, and the other continents;" and, further on, he adds:—"While, as has been pointed out, numerous illustrations can be adduced, shewing a larger oscillation over the same region with a high temperature and a dry atmosphere than with a low temperature and a moist atmosphere; the small summer oscillation on the coasts of the Mediterranean and those of the Atlantic adjoining, is in direct opposition to the idea that any such conclusion is general. For over those parts of the Mediterranean and Atlantic, the temperature is hottest in summer, and the air is driest—so dry, indeed, that no rain, or next to none, falls; and yet there, the amplitude of the oscillation now contracts to its annual minimum. On the western coasts of the Atlantic, from the Bahamas northwards to Newfoundland, the temperature is at the annual maximum, but the air is not dry, being liberally supplied with moisture, and the rainfall is generous. But with these very different meteorological conditions, there occurs, equally as in Southern Europe, a diminished oscillation during the summer months in the islands and near the coasts of North America; and, in the south of Europe, the

* Written in the beginning of March, 1876.

oscillation reaches its annual maximum, just at the season when the annual minimum occurs near the sea coasts, even although the general characteristics of the atmosphere be substantially the same in both cases."

I am not at present aware whether Mr. Buchan has been led by these observations to any definite conclusions as to the physical cause of the variation he so clearly summarizes in the passages above quoted. In the part of his memoir which has reached me, all theoretical discussion is deferred. But these passages afford such remarkable confirmation of an explanation at which I arrived some weeks since, on approaching the subject from an entirely different quarter, that I do not think it necessary to withhold longer the publication of my view. If Mr. Buchan's conclusions are the same as mine, the facts that I have to bring forward will serve to afford independent confirmation of that view.



Any person glancing over a series of curves illustrating the diurnal rise and fall of the barometer, cannot fail to be struck with the characteristic difference of those places with a continental and those with an insular climate. The case of the Mediterranean, described by Mr. Buchan, seems perhaps to be an exception; but, as I shall presently shew, it is an exception of such a kind as most strongly to confirm the rule. The accompanying curves are striking, perhaps extreme, examples of this characteristic difference. The first is that of Leh in Ladakh,* situated in the Indus valley (the observatory being 11,538 feet above the sea), and is for the month of September. The climate is characteristically dry, and the summer heat excessive, notwithstanding the elevation. The curve for Yarkand and Kashghar still further north and only 4,000 feet above the sea, is of similar character but smaller amplitude.† The second curve figured is that for the

* This is computed from the hourly observations recorded during six days by Captain E. Trotter, R. E., and of one day by Dr. J. Scully together with six days' observations by the latter at the hours of 4 and 10 A. M. and P. M.

† With respect to these curves however, see the final paragraph, page 328.

northern half of square 3 of the North Atlantic, published by the London Meteorological Office. In the former, the double oscillation has almost disappeared, the nocturnal fall of pressure being represented by little more than a halt for some hours between two periods of rising pressure; and nearly the whole fall of the day takes place between 9 A. M. and 5 P. M. In the case of the Atlantic curve, the day and night oscillations are almost exactly alike; the night oscillation being only slightly less than that of the day. These characteristic differences are perhaps best expressed by the ratio of the constant co-efficients U′ and U″ in Bessel's interpolation formula :—

$$x = M + U' \sin (n\,\theta + u')\ U'' \sin (n\,2\,\theta + u'') + \&c.$$

since the magnitude of U' determines the inequality; and that of U'', though variable under different conditions of climate, is so to a much less extent than the former term, and chiefly depends on the latitude. The following are the values of U' and U'' in English inches, and their ratios, for the mean diurnal curves of a few stations (chiefly Asiatic). The arcs u' u'' corresponding thereto are also given.

C		U′	u′	U″	u′	U′ U″
Yarkand	(9 months)	·0348	4° 33′	·0215	161° 59′	1·6 : 1
Leh	(September)	·0517	343° 9′	·0254	143° 19′	2· : 1
Lucknow	(Year.)	·0265	341° 30′	·0355	168° 53′	0·75 : 1
Hazaribagh	„	·0193	349° 46′	·0343	145° 45′	0·56 : 1
Calcutta	„	·0265	341° 24′	·0391	151° 7′	0·68 : 1
Bombay	„	·0179	337° 17′	·0385	157° 13′	0·46 : 1
Batavia	„	·0240	24° 7′	·0369	159° 34′	0·65 : 1
Square 3.	Atlantic	·0055	354° 51′	·0319	159° 26′	0·17 : 1

As a general rule, the more humid the station and the smaller the range of temperature, the smaller is the value of U'; and hence it has sometimes been spoken of as the temperature element of the oscillation; the double oscillation which is superimposed on it, being referred by Dove, Sabine, and Herschel to the varying tension of water vapour, by Lamont and Broun to some solar influence other than heat, and by Espy and Kreil to the oscillation of pressure produced by heat in an elastic fluid expanding and contracting under the influence of gravity. To me it seems that there can hardly be a doubt that the last explanation is the true one,* and

* True, that is to say, as thus stated in general terms. I do not however fully accept the detailed explanation afforded by any of these authors; and I am disposed to think that a more probable explanation of the morning oscillation is to be found in the retardation which the transmission of the exalted pressure of the lower to the higher strata must meet with, in the great thickness of highly attenuated but exceedingly cold air which constitutes these strata. This pressure cannot be transmitted

that this has not been generally recognized, I attribute to the fact that the consequences of the theory, as a purely physical problem, have never yet been traced out and verified by such a mass of facts as Mr. Buchan is now bringing together. So long as the *whole* phenomenon is not satisfactorily accounted for, some doubt may reasonably attach to the explanation offered of one only of its elements.

My own attention was first drawn to the subject of the explanation which I am about to give, by a paper of Mr. F. Chambers in the Philosophical Transactions for 1873, in which that gentleman showed as the result of an analysis of the diurnal variation of the winds at Bombay, that one element of this variation is a double rotation of the wind direction; of such a character, that the southerly components attain their maximum value at the epoch of the most rapid semi-diurnal rise of pressure, the easterly components at the epoch of maximum, the northerly with the most rapid fall, and the westerly with the epoch of minimum. On these facts Mr. Chambers based a suggested explanation of the barometric tides, regarding them as a phenomenon of static pressure; and assumed (as now appears, on insufficient grounds) that the phenomenon in the northern hemisphere is generally of the same type as at Bombay. There was indeed one feature in his explanation, which it seems difficult to reconcile with mechanical laws; since he supposed air to flow from both east and west towards a region where the pressure has already risen above the mean, and by its accumulation to produce a maximum of static pressure. But apart from this, the discovery was an important one; and, since it clearly shewed that a regular horizontal transfer of air corresponded to the oscillations of pressure, it held out a promise that further steps in the same path might clear up what appeared to be anomalous, and possibly lead to a complete explanation of the diurnal oscillation.

Some time before this paper reached me, the Rev. M. Lafont had placed in my hands four years' traces of a Secchi anemograph, erected on St. Xavier's College, Calcutta; and these having been measured off, tabu-

with a greater velocity than the sound wave, and it is probably much less; since the action being slow and prolonged, the heat developed by the compression must be in part dissipated. To explain the observed phenomenon on this hypothesis, the retardation must however be such, that the unrelieved excess of pressure at the ground surface, must be equal to that generated by from half to three quarters of an hour's action of the sun.

This would require us to assume a much lower average temperature for the higher strata than results from Pouillet's calculation, and also that a certain diurnal oscillation of temperature affects the atmosphere to a greater height then has been usually assumed. But this hypothesis is free from most of the objections to be urged against those of the authors quoted.—*Note added January 20th*, 1877.

lated, and reduced, I was interested to find that the diurnal wind variation at Calcutta showed the double diurnal oscillation quite as distinctly, and relatively even more prominently than that of Bombay. But one important difference presented itself. The north and south elements of the oscillation, while agreeing in epoch with those of Bombay, were reversed in direction; and, taken together with the latter, showed a tendency to a cyclonic circulation of the atmosphere around the peninsula during falling pressure, and an anticyclonic circulation with rising pressure. Moreover, the east and west components agreed almost exactly, in epoch, with the north and south components; the result being a movement of air from the north-west with falling pressure, and from the south-east with rising pressure. These facts, taken in conjunction with the positions of Bombay and Calcutta on opposite sides of the peninsula, seemed to point to the differential conditions of land and water as being probably concerned in the phenomenon. Another and not less important fact connecting the winds with the diurnal oscillation of the barometer appeared at the same time. When the wind variation was analysed by Bessel's method, there appeared an east and west oscillation of considerable magnitude, corresponding in epoch with the barometric inequality expressed by the first periodical term of the barometric formula. This was easily distinguished from the oscillation of the sea and land winds, since the latter are nearly north and south at Calcutta. At Bombay where the sea and land breezes are nearly east and west, such an oscillation would be undistinguishable, even if it really existed.

The east and west oscillation of diurnal period indicates an outflow of air to the eastward during the day time, an inflow from the east during the night; and the former phase of it evidently corresponds to the hot winds of the Gangetic plain and Northern India and, indeed, to the day winds of the dry months of the greater part of India. They blow towards the sea from the eastward, only in the western portion of the Dekhan, Mysore, &c. This system of day winds consists of an outflow of air from the peninsular towards the sea on both coasts, the westerly direction greatly predominating.

The next step in the enquiry was to ascertain what general cause would operate to produce this efflux and influx of air; and the obvious suggestion was that it must consist in the differential action of the sun's heat on dry air and water.

Let V be any volume of dry air at pressure P and absolute temperature T and let τ units of heat be communicated to it, raising its temperature from T to $T+t$, while the volume remains constant. The pressure will be increased thereby from P to P$+p$ wherein—

$$p = P\left(\frac{T+t}{T} - 1\right) = P\,\frac{t}{T} \;\ldots\ldots\ldots\ldots\; (1)$$

And $$\tau = V\rho \frac{P}{\mathrm{P}} \frac{\mathrm{To}}{T} tc \text{ ......} \quad (2)$$

Wherein ρ is the density of air at the standard pressure P and absolute temperature To and c its specific heat at constant volume, compared with water as unity.

If now the same quantity of heat τ be employed in evaporating water at temperature T, (the whole being consumed as latent heat) and filling the volume of air V with vapour at pressure p', the total pressure will become

P + p' and $$\tau = V\varsigma \frac{p'}{\mathrm{P}} \frac{\mathrm{To}}{T} \lambda$$

when ς is the hypothetical density of water vapour at P and To, and λ its latent heat at temperature T. Substituting for ς its approximate equivalent $\frac{5}{8}\rho$

$$\tau = V \tfrac{5}{8} \rho \; \frac{p'}{\mathrm{P}} \frac{\mathrm{To}}{T} \lambda \text{ ......} \quad (3)$$

and equating (2) and (3) and eliminating common factors,

$$p' = \frac{\mathrm{P} \; t \; c}{\frac{5}{8} \lambda} \text{ ......} \quad (4)$$

From (1) and (4).

$$p = p' \; \frac{5}{8} \; \frac{\lambda}{Tc} \text{ ......} \quad (5)$$

which gives the ratio of the increase of pressure produced by the same quantity of heat, employed in the one case simply in heating dry air, and in the other in charging it with vapour. At a temperature of 80° Fahr. = T = 541,

$$p = 7{\cdot}36 \, p'$$

that is to say, when a given quantity of heat is employed in heating dry air at the temperature of 80° it raises its pressure more than seven times as much as when it simply charges it with vapour without altering the temperature. With lower values of T the difference will be still greater.*

This great difference is no doubt much reduced in nature by the effects of radiation ; and while some evaporation is effected on the land surface, there is some increase of temperature over the sea : but it may be expected that some part of this difference will manifest itself in the greater intensity of the forenoon pressure in the lower strata of the atmosphere on the land as compared with the sea, and in fine clear weather as compared with cloudy weather, when banks of clouds present an evaporating surface. With

* Substituting for λ the general value determined by Regnault 1091·7 — ·695 (T — 493·2) the general expression for the ratio becomes

$$p = \left(\frac{5400{\cdot}87}{T} - 2{\cdot}61\right) p'$$

regard to this latter point, it has been shewn by Lamont and Kreil's investigations, that between clear and cloudy days there is a difference of this kind; and that it is manifested, not only in the greater magnitude of the diurnal coefficient u', but also, although to a much less degree, in that of the semidiurnal coefficient u'' of the barometric formula. Further evidence of the same kind is afforded by the values of these coefficients for the several months at Calcutta.

	U'	u'	U''	u''
January, ...	·0287	330° 18′	·0415	151° 34′
February, ...	·0319	327° 12′	·0423	146° 48′
March, ...	·0343	329° 27′	·0437	146° 44′
April, ...	·0361	336° 53′	·0425	146° 38′
May, ...	·0325	344° 43′	·0385	148° 13′
June, ...	·0218	357° 28′	·0336	146° 23′
July, ...	·0192	2° 6′	·0396	150° 30′
August, ...	·0218	0° 5′	·0372	144° 29′
September, ...	·0232	354° 41′	·0400	151° 25′
October, ...	·0234	343° 12′	·0393	160° 59′
November, ...	·0250	337° 38′	·0399	164° 22′
December, ...	·0270	335° 18′	·0411	158° 55′

The driest months in Northern India being March and April, while July is the wettest and most cloudy.

On Espy and Kreil's hypothesis of the cause of the double oscillation, there is no apparent reason why the evening maximum, arising from contraction and dynamic pressure, should be equal to the morning maximum; which seems unquestionably due to the increased tension of the lower atmosphere, in consequence of heating and the introduction of vapour; and any inequality will of course appear in the value of u' or of the coefficients of other terms of odd periodicity. But the fact established by the anemometer, that an outflow of air from a heated land area takes place during the day time, at once assigns a cause for the greater part of the equality, *viz.* an alteration of the static pressure. This is not an overflow in the upper regions of the atmosphere, but an outflow of the lower strata* or a tendency in that direction. It does not, of course, follow that, to produce a reduction in the mass of air over a contiment, there should be an actual motion of the air outwards in all directions. The very small forces in action will be manifested even more in retarding inflowing currents than in accelerating efflux; and it is only in very dry and highly heated region such as India, that they produce well marked diurnal surface winds blowing outwards towards the sea; winds of elastic expansion, such as are the hot

* Excepting of course in the immediate neighbourhood of the coast, when the sea breeze of the lowest stratum is a secondary effect of the outflow.

winds of India and Australia, winds which are distinct from convection currents, though, it may be, coexisting with and accelerating them. The relation of these winds to the barometric tides is very marked, but it does not seem that the differences of tidal pressure would suffice to generate them, were there not a movement of the air in the same direction, arising from more persistent differences of pressure. They probably also depend much on local and irregular differences of pressure.

The air thus removed in the day time from continental areas must collect over the nearest areas of evaporation, with the effect of diminishing the midday fall of pressure over those tracts; and thus seem to be explained those apparent anomalies in the magnitude of the midday semi-oscillation of the barometer, to which, in the passages quoted from Mr. Buchan's memoir, he has drawn attention; *viz.* in the case of the Mediterranean area and the Atlantic coast of North America.

The direction in which this movement of the air takes place will of course vary with the locality; but there will always be, on an average, a greater diurnal movement towards east coasts than towards those facing to the west. This may be illustrated by the case of Calcutta and Bombay, and it is more extensively illustrated by the predominant westerly direction of the land winds of India, and the cold westerly diurnal winds* that blow across the high plains (17,000 to 19,000 feet) of the Changchenmo and Rupshu in Western Tibet. The reason is sufficiently obvious. As the great semi-diurnal waves of pressure advance from East to West, the local barometric gradient of any place (in so far as it is determined by the diurnal oscillation) will be expressed by a tangent to the existing phase of the wave. During the hottest part of the day, *viz.*, from 9 or half-past 9 to half-past 4 or 5, this gradient (which is the steepest and most prolonged of the four) inclines to the eastward and increases the declivity towards east coasts arising from the excess of pressure over the land. In the opposite direction, *viz.*, towards west coasts, it goes to diminish that declivity. At night the case is reversed. The west to east barometric gradient, from 10 P. M. to half-past 3 or 4 A. M. is in the same direction at that tending to produce an influx of air from the sea towards the land on west coasts; this however is opposed to the land wind of the coast line, which is a true convection current and arises from quite different causes; and although traceable in the wind variation at Bombay, it there manifests itself only by decreasing the velocity of the former. There are moreover independent grounds for the influence that this compensating inflow chiefly affects the higher strata of the atmosphere, while the day wind is felt in the lower and more heated strata. At Calcutta the easterly (or negative westerly)

* This I state on the authority of Dr. Cayley who assures me that on the high plains these afternoon winds are always from the West.

tendency of the wind at night is very prominently exhibited in the curve of diurnal variation ; but, although of longer duration, it is at no time so intense as the westerly tendency in the early afternoon hours.

In like manner may be explained the difference of epoch of the corresponding phases of the semi-diurnal East and West variation at Calcutta and Bombay. The gradient of pressure, in so far as it depends on the semi-diurnal oscillation, will of course be to the West with a rising pressure and to the East with a falling pressure, and this normal tidal gradient is affected by the small difference of amplitude over land and sea, in such manner, that its changes will be accelerated as affecting East coasts and retarded as affecting West coasts. Now if we suppose that the acceleration in the one case and the retardation in the other amount to an hour or an hour and a half, and that the interval between the change in the direction of the gradients, and their effects on the wind as manifested by the anemometer, is also about an hour and a half, we should roughly reproduce the conditions shewn to exist at Calcutta and Bombay respectively.

According to this view, the local static pressure of the atmosphere, except in so far as it is affected by irregular movements, is shewn by the height of the barometer at the hours of minimum pressure, and the difference of these expresses the weight of the atmosphere removed and restored by the oscillatory movements between land and sea.

There is much reason to believe that an oscillation of a similar character takes place between low plains and deep valleys on the one hand and mountain masses on the other, the air being transferred from the low plains and valleys to the hill masses and high plains (such as those of Tibet) in the day time and returned during the night. Thus, it seems to me, are to be explained the very great diurnal oscillations of the Leh barometric curve, the great amplitude of the midday tide at stations in the Assam valley, and the diminished tide at places such as Roorkee and Lahore which lie near the hills on the margin of broad plains. Also the stormy afternoon winds of the Dipsang, Changchenmo and Rúpshu plains, and the fact that at hill stations such as Simla and Darjiling, the night barometric tide exceeds in amplitude that of the day.